

Population Inversion

Laser theory is dealing with a system that contains a large number of active atoms or molecules that interact with electromagnetic radiation. Normally, the system (substance or active medium) absorbs electromagnetic radiation incident on it. While in the process of obtaining the laser beam must make the medium as an amplifier to electromagnetic radiation incident on it. This can be done by obtaining the state of population inversion (reverse distribution) by disrupting the natural state of the distribution of atoms or molecule resulting from the state of thermal equilibrium. By population inversion we mean getting the highest number of excited atoms at a high energy level. Figure 5.8 shows an illustrative diagram of the state of natural distribution of a particular material's atoms at a range of different energy levels at a given temperature

Boltzmann Distribution

If we suppose an isolated system consisting of a number of N atoms or molecules are distributed over a number of energy levels at a certain temperature. We assume that N_1 of these atoms or molecules occupy the first energy level with amount of energy E_1 , and the number of N_2 in the energy level E_2 . The Boltzmann equation gives us the relative distribution (N_2 / N_1) of all atoms or molecules of two energy levels E_2 compared with E_1 :

$$N_2/N_1 = \exp \left(-(E_2 - E_1)/kT \right)$$

k = Boltzmann constant 1.38×10^{-23} Joule / OK

T = temperature in Kelvin degrees. ($^{\circ}\text{K}$) absolute temperature

Boltzmann's distribution in the perspective of physics and mathematics is a function of the probability of the distribution of a system of particles (atoms or molecules) at energy levels, that is, it gives the probability of a certain number of atoms or molecules at any level energy ($i = 1, 2, 3, \dots$). Boltzmann equation shows the dependence of the number of atoms or molecules (N_i) on the energy level (E_i) at the temperature T , and from this equation we see that:

1. The higher the temperature, the greater the distribution of atoms or molecules.
2. The higher the level of energy, the lower the distribution number.

The relationship between the relative distribution number (N_2 / N_1) does not depend on the values of E_1 and E_2 energy levels, but only on the difference between them. $E_2 - E_1$. The figure below shows the distribution of atoms or molecules at each energy level in the case thermal equilibrium condition. Figure 5.8 shows an illustrative diagram of the energy state of natural distribution of the atoms or molecules of a given material in a range of different energy levels in the case of a thermal equilibrium.

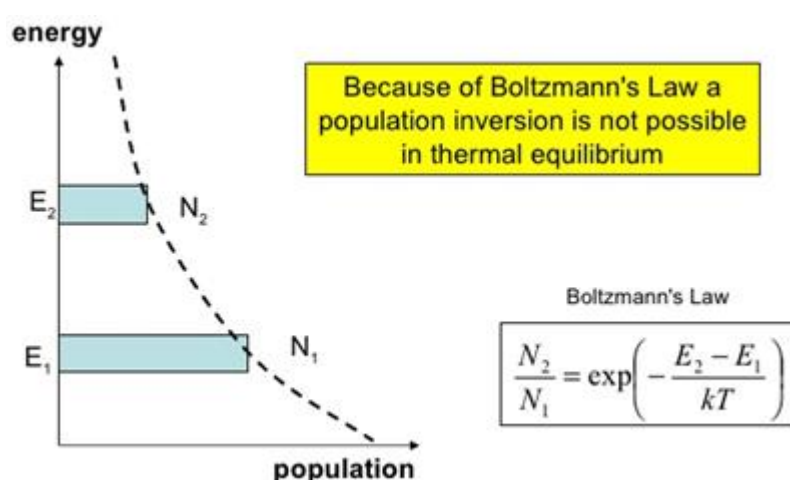


Figure 5.7: Normal population distribution

in this figure Lengths of horizontal lines represents the number (population) of atoms at each level of the energy states. Where we note that the majority of atoms always in the ground energy level, and the population of atoms in the upper energy level lower than any population in the energy state with less energy and so sequentially. The absorption coefficient of the material depends on the wavelength of light corresponding to the transition of the atoms of the material from E_3 to E_2 and It is proportional to the quantity of $N_2 - N_3$. If there is a large difference in the population of the number of atoms for the two energy states, a greater absorption coefficient is produced. If the atoms are transferred from the E_2 level and added to the E_3 level, the difference in the population of the number of atoms will be reduced. As a result, the absorption coefficient is reduced.

If we assume that there are two levels of energy, the distribution of the number of atoms is equal in these state, in such case the absorption coefficient will be zero, although absorption is still occurring. The

process of the stimulated emission continues to operate at a rate similar to the absorption process, where the emission photons replace the photons removed by absorption. There is a balance between the emission and absorption process, and the difference between them is equal to zero. As a result, the value of the effective absorption coefficient is zero.

Figure 5.9, shows the numerical population of the atoms in the energy states E_2 and E_3 has been altered so that the quantity $N_2 - N_3$ become negative.

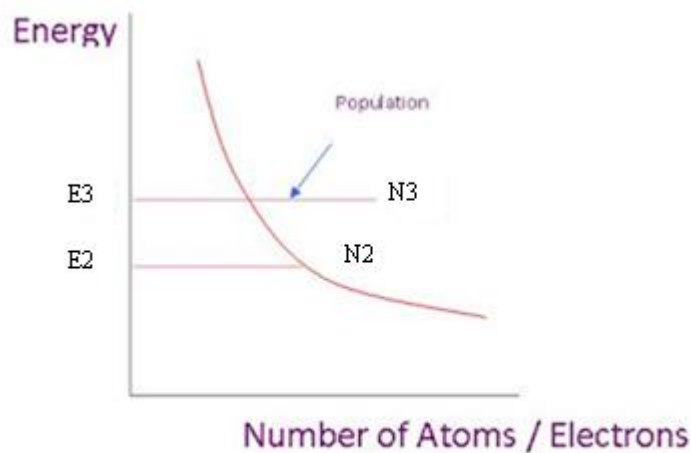


Figure 5.8: Population inversion between states E_2 and E_3

One of the basic requirements for the case of population inversion between two energy levels such as E_2 and E_3 is that the absorption coefficient must become negative. population inversion is the term given when a group of atoms is distributed at two levels of energy. The distribution of these atoms inverted with respect to each other. When the population inversion occurs, then the absorption coefficient is in the equation $= e (-kx)$ Has a negative value; in this case the transmission value must be greater than one. The appropriate circumstance arises for the stimulated emission process to occur at a rate greater than the absorption rate. In these conditions, amplification of the light passing through the active material is obtained.

The population inversion between E_2 and E_3 now requires that the value of the absorption coefficient also must become negative. This condition is called a "population inversion" because the populations of the two atomic

states have been inverted with respect to each other. When a population inversion exists, the absorption coefficient in the equation $T = e^{-kx}$ is negative; and the transmission is then necessarily greater than one. Under these conditions, stimulated emission occurs at a rate greater than the rate of absorption, and the light passing through the material is amplified.

When a one color parallel beam passes through the material (or can be expressed active medium of the laser). This medium will absorb part of the beam. Or by amplify the light by providing additional optical photons, which means that the medium increases or decreases the optical intensity of the beam, depending on the number of atoms that are in their excited state and the number of atoms in the lower state,

The gain to amplify the light of the beam passing through the active material depends on the amount $N_3 - N_2$, which means that the increase in the population inversion leads to greater optical gain. To increase the optical gain of the generated laser ray, some appropriate methods must be adopted to increase the number of atoms in the upper laser energy state to obtain the transition that generates the laser ray. As the laser process itself generates the emission of radiation as a result of the transfer of atoms from the upper energy levels to the lowest energy levels, that is, the transition to the lower state leading to the loss of energy by radiation from the system, in this case there should be some ways to reduce the number of increased atoms as a result Laser action from the low power lasing state. If atoms are allowed to remain at the low energy state, the number of atoms at this state will increase to the extent that there is no population inversion between the two states, and then the laser process (the emission process leading to energy loss from the active medium) will stop.

Rate Equations for Spontaneous Emission

For simplicity we shall assume:

1. The material is composed of many identical atoms.
2. Each atom has only two energy levels: E_1 and E_2 .
3. The only decay mechanism of energy level E_2 is spontaneous emission.
4. At time t , N_1 atoms are in energy level E_1 , and N_2 atoms in energy level E_2 .

The rate at which the excited atom population $N_2(t)$ decays from the higher energy level (E_2) to the lower energy level (E_1), by stimulated emission is given by the decay coefficient g_{21} multiplied by the instantaneous population number of this level $N_2(t)$ as seen by the first order differential equation:

$$\frac{dN_2(t)}{dt} = -g_{21}N_2(t) = -N_2(t)/\tau_2$$

This equation defines the lifetime τ_2 of energy level E_2 :

$$\tau_2 = 1/g_{21}$$

The solution to the rate (differential) equation is:

$$N_2(t) = N_2(0)e^{-g_{21}t} = N_2(0)e^{-t/\tau_2} \quad \dots 5.5$$

If at a specific moment ($t = 0$) the number of atoms in the excited state E_2 is $N_2(0)$, then when we leave the system without external influence, The number of atoms in the excited energy level will be dissipated exponentially according to the equation 5.5. Pay attention that in spontaneous emission, the population number (N_1) of the lower energy level (E_1) is unimportant

Stimulated Transitions

The strength of an optical signal (The number of photons) is described by:

- i. *Intensity* (I), which means the ratio of measured power over a specific surface area (since power is a measure of the amount of energy over time, than Intensity is a measure of the amount of energy over time over surface area).
- ii. *Energy density* ($n(t)$), The number of photons per unit volume = The energy of the electromagnetic radiation in units of $h\nu$ or $\hbar\omega$.

An Optical signal is an oscillating electromagnetic field, and an atom can be described as an electric dipole. When an optical signal of the right frequency ($h\nu$ equal to the energy difference between the two energy states ($E_2 - E_1$)) is approaching an atom, both the atoms at the lower energy state (E_1) and the atoms at upper energy state (E_2) will start to oscillate. That is why there are two forced processes: absorption and stimulated emission.

Absorption Rate Equations

The incident light signal (photons) causes the atoms to "jump" from the lower energy level (E_1) to the upper energy level (E_2). The absorption rate is proportional to the product of the density $n(t)$ of the incoming photons (the number of photons per unit volume) with the number of atoms ($N_1(t)$) in the lower energy level (E_1):

$$\frac{dN_2(t)}{dt} = K \cdot n(t) \cdot N_1(t) \quad \dots 5.6$$

Each photon excites one atom to the higher energy level.

K = Proportionality constant, Relative measure of the response strength of the atom to the radiation falling on it in this specific transition.

5.9 Stimulated Emission Rate Equations

The incoming light (photons) causes the oscillation of atoms (forced oscillations) in the upper energy level (E_2), and make a transition to the

lower energy level (E_1). In this process two photons are emitted together: The incoming photon and the photon from the energy transition $h\nu = E_2 - E_1$. The stimulated emission rate is proportional to the product of the energy density $n(t)$ of the incoming photons (the number of photons in a unit volume) with the number of atoms ($N_2(t)$) in the upper energy level (E_2):

$$\frac{dN_2(t)}{dt} = K \cdot n(t) \cdot N_2(t)$$

5.10 Proportionality Constant (K)

From quantum considerations, we get that the proportionality constant (K) for stimulated emission and (stimulated) absorption are identical. This constant depends on the frequency (ν) of the incoming photon.

The value of K is maximum when the frequency of the incoming photon is equal to the transition frequency ν_{21} . The farther apart from the transition frequency the less the value of the proportionality constant up to zero. Every transition has a linewidth ($\Delta\nu$) around the transition frequency.

This linewidth shows the frequency range in which transitions can occur. If the frequency of the incoming photon is not in the range $\nu_{21} \pm \Delta\nu$, then the value of K is zero.

Diagram of Energy Level Population

We shall summarize all the transitions in the diagram of energy level population (Figure 5.11).

The rate equation for the population level E_2 summarizes the spontaneous emission and the two stimulated emissions, for the simple case of a two level system:

$$\frac{dN_2(t)}{dt_{total}} = \frac{dN_2(t)}{dt_{absorp}} + \frac{dN_2(t)}{dt_{stimul}} + \frac{dN_2(t)}{dt_{spont}}$$

$$\frac{dN_2(t)}{dt_{total}} = Kn(t)[N_1(t) - N_2(t)] - g_{21}N_2(t) = \frac{dN_1(t)}{dt_{total}}$$

We should remember that the spontaneous emission and the stimulated emission occur at the same time, and are independent of each other, so their emission rates can be added. The stimulated emission process is the result of a signal that forces the atom to respond to the resonance emission, oscillating at the same frequency, and being temporary and spatially coherent (with the same phase and amplitude). On the other end, the spontaneous emission is in all directions in space, and each photon is randomly emitted.

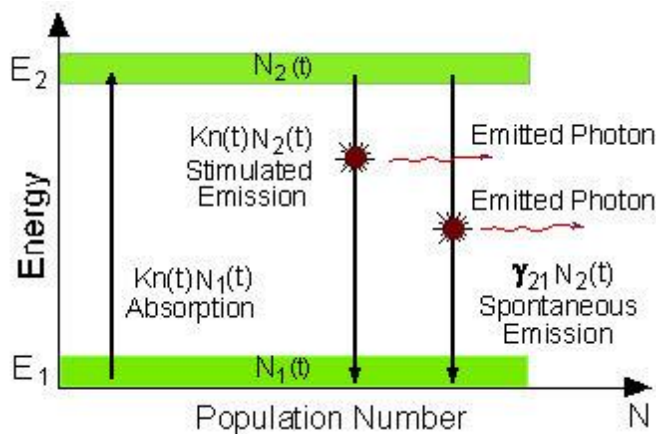


Figure 5.11: Schematic Energy Level Diagram

5.11 Amplification

We saw in the rate equation 5.8 that:

- Two processes decrease the population number of the excited level: Spontaneous emission and stimulated emission.
- One process increases the population number of the excited state - absorption.

Since the same amount of energy ($h\nu$) is involved in every transition up or down, then the rate at which energy is absorbed in a unit volume of atoms is given by the transition rate times the unit of energy:

$$\frac{dU_a}{dt} = Kn(t)[N_1(t) - N_2(t)] \times h\nu \quad \dots 5.9$$

U_a = energy density in stimulated transitions.

The incoming energy is:

$$U_{signal}(t) = n(t) \times h\nu$$

The absorbed energy is taken out of the incoming signal, so the rate of loss of energy from the incoming signal is:

$$\frac{dU_{signal}}{dt} = -Kn(t)[N_1(t) - N_2(t)] \times h\nu$$

$$\frac{dU_{signal}}{dt} = -K[N_1(t) - N_2(t)] \times U_{signal} \quad \dots 5.11$$

The same equation can be written for the photon density:

$$\frac{dn(t)}{dt} = -Kn(t)[N_1(t) - N_2(t)] \quad \dots 5.12$$

Absorption or Amplification:

From the equation of rate of loss of energy from the incoming signal, we see that the sign of the difference in the population number ($\Delta N = N_1(t) - N_2(t)$) determines if the energy density of the incoming signal will increase or decrease as a function of time. Consider two possible situations:

1. When a collection of atoms is in a normal population (Thermal equilibrium), the population number of the higher energy state (E_2) is lower than The population number of the lower energy state (E_1):

$$N_1(t) > N_2(t).$$

In this situation, the only absorption is possible, and as a result, the atoms will receive energy from the incoming signal which will decrease in magnitude.

2. When a collection of atoms is in a population inversion, the sign in front of the parenthesis is minus, and the signal will increase - Amplification!

Energy from the system of atoms will be transferred to the incoming signal, and amplify it at a rate which is proportional to the difference in population numbers and to the intensity of the incoming signal.

If the material is in a state of thermal equilibrium, absorption is the only process that occurs and the amplification process does not occur. To produce amplification, the material must be in state of population inversion where more atoms are pumped to the state of energy that is excited compared to the lower energy level.

After understanding the amplification process at the microscopic level of atoms, we shall connect this process to the macroscopic system, as we did in the process of absorption of electromagnetic radiation.

In the process of absorption in the laser medium, the absorption coefficient (α) depends on the material and on the difference in population numbers ($\Delta N = N_1(t) - N_2(t)$) between the energy states E_1 and E_2 , as:

$$\alpha = K[N_1 - N_2] \quad \dots 5.13$$

The proportionality constant (K) depends on the material and the wavelength of the laser radiation. As long as $N_1(t) > N_2(t)$, α is positive, and the process is absorption.

In the "population inversion" situation, $N_1(t) - N_2(t) < 0$, so that $N_2(t) > N_1(t)$, then α is negative. According to Lambert law: $I = I_0 e^{-\alpha x}$, the factor $(-\alpha x)$ is positive, which means that, the intensity at the output (I) is bigger than the intensity at the input (I_0), thus amplification (gain of energy). In case of amplification, K is called Gain Coefficient.

The probability of the stimulated transition by the incoming radiation is identical for the two processes (absorption and stimulated emission). The direction in which more processes will occur depends on the population of the energy levels at that moment.

EXAMPLE: DEPENDENCE OF AMPLIFICATION ON THE LENGTH OF THE LASER.

A laser is 15 cm long. For a certain wavelength, the amplification of the laser is 1.5. Calculate the amplification of this laser if the length of the active medium is 30 cm.

Solution:

Using the definition of amplification:

$$\text{amplification} = \frac{I}{I_o} = e^{-\alpha x}$$

Substitute the known parameters:

$$1.5 = e^{-\alpha \cdot 15}$$

$$\alpha = -0.027 \text{ cm}^{-1}$$

Using the new length in the amplification definition:

$$\text{amplification} = \frac{I}{I_o} = e^{-\alpha x} = e^{-(-0.027 \times 30)} = 2.25$$

Whenever there is an increase in the length of the active medium there is an increase in the amplification.

There is a limit to this conclusion. In the above discussion absorption in the active medium was not taken into account. Later we shall see how to calculate both absorption and amplification. An example of such mathematical calculations is:

- Radioactive decay chain.
- Chain of water tanks in which the water is flowing from the higher to the lowest through all the tanks.

If the absorption rate is proportional to N_1 , and the emission rate is proportional to N_2 , with the same proportionality factor, then the number of photons in the output laser beam, depends on $N_1 - N_2$.

Three-Level Laser

Figure 5.12 shows a diagram of the energy state of a three-level laser system. Two states of energy are responsible for the emission of the laser beam: low laser energy level (E_1), and upper laser energy level (E_2). To simplify interpretation, we ignore spontaneous emissions.

In order to obtain the stimulated emission and to achieve the lasing process, the population inversion mode must be created, so the system must be pumped to form more atoms at the E_2 state, so that it is more than the E_1 energy state. Atoms must be pumped from ground state (E_1) to energy state E_3 , whose average life time is $(10^{-7} \text{ } 10^{-8})$ seconds. The

atoms stay for this period and then usually decay by a non-radioactive transition to the E_2 energy state, which is long lifetime. (meta-stable).

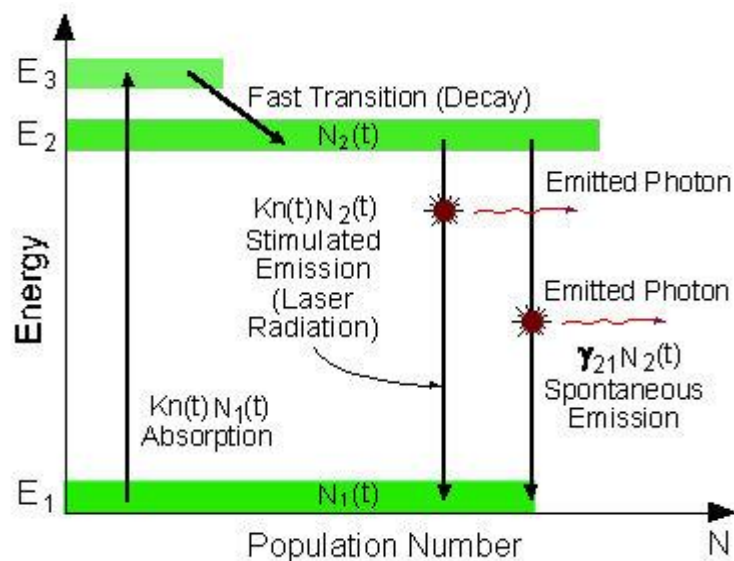


Figure 5.12: Energy level diagram in a three-level laser

since the E_2 is relatively long lifetime meta-stable energy state (10^{-3} seconds long, there are many atoms at this level) If the pump is strong enough to pump more than 50% of the atoms it will be in E_2 , population inversion occur, , the lasing process can occur.

Four-Level Laser

in Figure 5.13 shows a schematic of the laser energy states diagram of four levels. Compared with thee equivalent three-level laser, there is an additional energy state above the ground energy state. This additional state has a very short lifetime. The pumping process in a laser of four levels is similar to a three-level laser pumping process. This is achieved by the rapid increase in the population of atoms in the upper laser energy state (E_3) through the decay of the high energy level (E_4). The advantage of the fourth level laser is the lower the population of atoms at the low laser energy state (E_2). To create a population inversion state, there is no need to pump more than 50% of the atoms to the E_3 laser state.

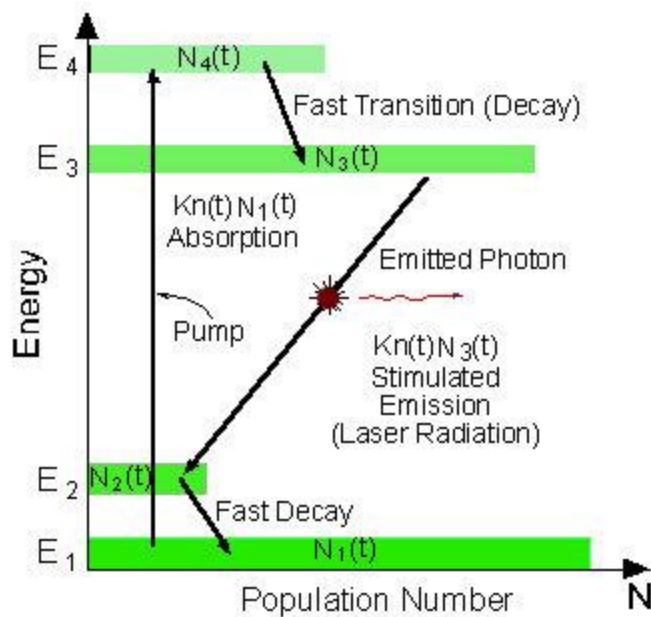


Figure 5.13: Energy level diagram in a four level laser

The number of atoms at the lower laser level ($N_2(t)$) decreases as a result of rapid decay to the ground energy level, so it is virtually empty. Thus, the continuous operation of the four laser levels is possible even if 99% of the atoms remain in the ground energy level. Advantages of Laser Four Levels Compared to Laser Three Levels:

- The threshold for obtaining the stimulated emission in the four levels laser is lower.
- Be more efficient.
- The required pumping rate for population inversion is lower.
- It is possible to work or operate in continues mode.